Euler Coordinates

The Euler coordinate system is introduced here as an orthogonal coordinate system in which the Euler line is x-axis and the orthic axis is y-axis, and the origin is X(468). The positive direction on the x-axis is that of the vector X(468)-to-X(30), the positive direction of y-axis is that of the vector X(468)-to-X(523).
If P=(u, v, w) where u, v and w are the normalized barycentrics or u+v+w=1, then the Euler coordinates (x_e, y_e) for P are defined by

$$x_e = S_Au+S_Bv+S_Cw$$
$$y_e = S_A(S_B-S_C)u+S_B(S_C-S_A)v+S_C(S_A-S_B)w,$$

Here, x_e = kd and y_e = kSd', where $k=(E-8F)^{1/2}=6|X(2)X(3)|$, and d and d' are the signed distance from P to the orthic axis and to the Euler line, respectively.
For example, for $P_1 =(S_B, S_C, S_A)/N$, the normalization constant $N = S_A+S_B+S_C = (E+F)$, the Euler coordinates are given by

$$x_e(P_1)=S^2/(E+F),$$
$$y_e(P_1) = (S_AS_B^2+S_BS_C^2+S_CS_A^2-3FS^2)/(E+F),$$

Similarly, for $P_2 = (S_C, S_A, S_B)/N$, $N= S_A+S_B+S_C = (E+F)$, the Euler coordinates are given by

$$x_e(P_2)=S^2/(E+F),$$
$$y_e(P_2)=(-S_A^2S_B-S_B^2S_C-S_C^2S_A+3FS^2)/(E+F).$$

Since the triangle center is indexed as X(n) = (x : y : z) in ETC, where x, y, z are the unnormalized barycentrics, P(u, v, w) is given by the normalized X(n) = (u(n), v(n), w(n), where u(n) = x/N, v(n) = y/N, w(n) = z/N, N=x+y+z. For example, the centroid is given as X(2) = (1 : 1 : 1), then the normalized X(2) is given as X(2) = (1, 1, 1)/3, and the Euler coordinates for X(2) is given by $x_e(2) = (E+F)/3$, $y_e(2) = 0$ (these notations are used hereafter). Similarly, the orthocenter X(4) is given as X(4) = (S_BS_C ::) in ETC, where " :: " means that v(4) and w(4) are obtained by the cyclic permutation of u(4) = S_BS_C. The normalized X(4) is given as X(4) = $(S_BS_C, S_CS_A, S_AS_B)/S^2$, since $N = S_BS_C + S_CS_A+S_AS_B =S^2$, then the Euler coordinates for X(4) are given by $x_e(4) = 3FS^2/S^2 = 3F$, $y_e(3) = 0$. The triangle center X(30)=X(2)-X(4) = $(S^2-3S_BS_C, S^2-3S_CS_A, S^2-3S_AS_B)/(3S^2)$, then the Euler coordinates for X(30) are given by $x_e(30) = x_e(2) - x_e(4) = (E+F)/3 - 3F = (E-8F)/3$, $y_e(30) = 0$. Similarly, the triangle center X(523) is given as $X(523) = P_1 - P_2 = (S_B-S_C, S_C-S_A, S_A-S_B)/(E+F)$ (given above), the Euler coordinates for X(523) are given by $x_e(523) = x_e(P_1) - x_e(P_2) = 0$ and $y_e(523) = y_e(P_1) - y_e(P_2) = \{S_AS_B(S_B+S_C)+S_BS_C(S_C+S_A) +S_CS_A(S_A+S_B) - 6FS^2\}/(E+F) = (E-8F)S^2/(E+F)$. The triangle center X(468) is given by $X(468) = (-3FS^2 +(E+F)S_BS_C ::)$ and $N = -9FS^2+(E+F)S^2 = (E-8F)S^2$, then the Euler coordinates for X(468) are given by $x_e(468) = \{-3(E+F)FS^2+3(E+F)FS^2\}/\{(E-8F)S^2\} = 0$, $y_e(468) = 0$. The Euler coordinates (x_e, y_e) for the main triangle centers are summarized in Table 1.

Table 1 Barycentrics and Euler coordinates ($x_e(n)$, $y_e(n)$ for triangle centers X(n)

X(n)	Barycentrics	$x_e(n)$	$y_e(n)$
X(1)	(0 : 0 : a)	$\$aS_A\$/\$a\$$	$\$aS_A(S_B-S_C)/\$a\$$
X(2)	(1 : 1 : 1)	$(E+F)/3$	0
X(3)	$(S^2-S_BS_C ::)$	$(E-2F)/2$	0
X(4)	$(S_BS_C::)/S^2$	3F	0
X(5)	$(S^2+S_BS_C ::)$	$(E+4F)/4$	0
X(6)	$(S_B+S_C ::)$	$S^2/(E+F)$	$-\$S_A^2(S_B-S_C)\$/\{2(E+F)\}$
X(23)	$(-(E+4F)S^2-4(E+F)S_BS_C ::)$	$-(E+F)$	0
X(30)	$(S^2-3S_BS_C::)$	$(E-8F)/3$	0
X(74)	$(2(E-2F)S^2-3(E-2F)S_BS_C$ $-2S_AS^2::)$	$-\{3(3E-4F)F-4S^2\}$ $/(E-8F)$	$-2\$S_A^2(S_B-S_C)\$/(E-8F)$
X(230)	$((E+F)^2-3S^2+S_BS_C - S_A^2::)$	0	$-(E+F)\$S_A^2(S_B-S_C)\$/\{2\{(E+F)^2$ $-3S^2\}$
X(468)	$(-3FS^2+(E+F)S_BS_C ::)$	0	0
X(523)	$(S_B-S_C ::)$	0	$(E-8F)S^2/(E+F)$

The following relations are useful to calculate $y_e(n)$,

$$\$S_A^{k+2}(S_B-S_C)\$ = f_k(E, F, S)\$S_A^2(S_B-S_C)\$, k = 1, 2, 3\ldots,.$$

where $\$S_A^2(S_B-S_C)\$ = -(S_B-S_C)(S_C-S_A)(S_A-S_B)$ and $f_k(E, F, S)$ are given by

$$f_1 = (E+F),\ f_2 = (E+F)^2-S^2,\ f_3 = (E+F)^3+(2E+F)S^2,\ f_4 = (E+F)^4-(E+F)(3E+F)S^2+S^4,\ldots\ldots$$

Further, $\$aS_A(S_B-S_C)\$$ in Table 1 is related to $\$S_A^2(S_B-S_C)\$$ as below

$$\$aS_A(S_B-S_C)\$ = -[\{(E+F)+\$ab\$\}/\{(E+F)\$a\$+\$aS_A\$+2abc\}]\$S_A^2(S_B-S_C)\$.$$

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From the definition of the Euler coordinates, P(u, v, w) is given in terms of the Euler coordinates (x_e, y_e) as

$$u = [-3FS^2+(E+F)S_BS_C+(S^2-3S_BS_C)xe+(S_B-S_C)ye]/\{(E-8F)S^2\},$$
$$v = [-3FS^2+(E+F)S_CS_A+(S^2-3S_CS_A)xe+(S_C-S_A)ye]/\{(E-8F)S^2\},$$
$$w = [-3FS^2+(E+F)S_AS_B+(S^2-3S_AS_B)xe+(S_A-S_B)ye]/\{(E-8F)S^2\}.$$

If the triangle center X = (u, v, w) is on the Euler line or ye = 0, then

$$u = [-3FS^2+(E+F)S_BS_C+(S^2-3S_BS_C)xe]/(E-8F)S^2 = GS^2+HS_BS_C$$
$$v = [-3FS^2+(E+F)S_CS_A+(S^2-3S_CS_A)xe]/(E-8F)S^2 = GS^2+HS_CS_A$$
$$w = [-3FS^2+(E+F)S_AS_B+(S^2-3S_AS_B)xe]/(E-8F)S^2 = GS^2+HS_AS_B$$

where G and H are the Shinagawa coefficients (S-coefficients) and are given as

$$G = [-3F + xe]/(E-8F)$$
$$H = [(E+F) - 3xe]/(E-8F).$$

For example, from Table 1, xe = (E+F)/3 for X(2), then G and H are given by

$$G = \{-3F+(E+F)/3\}/(E-8F) = 1/3$$
$$H = \{(E+F) - (E+F)\}S^2 = 0,$$

which is equivalent to the S-coefficients (1, 0).
If the triangle center X = (u, v, w) is on the orthic axis or $x_e = 0$, then

$$u = \{-3FS^2+(E+F)S_BS_C+ (S_B-S_C)ye\}/\{(E-8F)S^2\}$$
$$v = \{-3FS^2+(E+F)S_CS_A+ (S_C-S_A)ye\}/\{(E-8F)S^2\}$$
$$w= \{-3FS^2+(E+F)S_AS_B+ (S_A-S_B)ye\}/\{(E-8F)S^2\}.$$

For example, for the triangle center X(230) on the orthic axis in Table 1, $x_e = 0$ and y_e is given by

$$y_e(230) = -(E+F)\$S_A{}^2(S_B-S_C)\$/\{2\{(E+F)^2-3S^2\}$$

then

$$u(230) = [\{-3FS^2+(E+F)S_BS_C\}\{2\{(E+F)^2-3S^2\}-(E+F)(S_B-S_C)\$S_A{}^2(S_B-S_C)\$]$$
$$/[\{(E-8F)S^2\}\{2\{(E+F)^2-3S^2\}\}$$
$$= [\{(E+F)^2-3S^2\}S^2+S^2S_BS_C-S_A{}^2S^2]/[2\{(E+F)^2-3S^2\}],.$$

where the following relation is used,

$$(S_B-S_C)\$S_A{}^2(S_B-S_C)\$ = -(E+F)^2S^2+3(E+F)FS^2+3S^4+2(E+F)^2S_BS_C-9S_AFS^2+S_AS^2$$

If the triangle centers is not on the Euler line, the normalized X(n) = (u(n), v(n), w(n)) is often given as

$$u(n) = [GS^2 + HS_BS_C + \Sigma c_kS_A{}^k]/N, \quad N=\{(3G+H)S^2+\Sigma c_k\$S_A{}^k\$\}$$

and v(n), w(n) are obtained by the cyclic permutation of u(n), where c_k are constants. Then the Euler coordinates are given by

$$x_e = [\{(E+F)G+3F\}S^2+\Sigma c_k\$S_A{}^{k+1}\$\}/N,$$
$$y_e = \Sigma c_k\$S_A{}^{k+1}(S_B-S_C)\$/N.$$

For example, X(74) in Table 1 is given by

$$u(74)=[2(E-2F)S^2-3(E-2F)S_BS_C-2S_AS^2]/(E-8F)S^2,$$

then the Euler coordinates for X(74) are given by

$$x_e(74) =\{2(E+F)(E-2F)+3(-3E+6F)-2(E+F)^2+4S^2\}/(E-8F)$$
$$=-\{3(5E-4F)F-4S^2\}/(E-8F),$$
$$y_e(74) = -2\$S_A{}^2(S_B-S_C)\$/(E-8F).$$

The following three Tables are summarized the Euler coordinates ($x_e(n)$, $y_e(n)$) for the triangle centers X(n) in ETC, whose normalization constants N are proportional to S^2 or $(E-8F)S^2$.

Table 2 shows the Euler coordinates (xe, 0) for the triangle center X(n) on the Euler line which has a constant S-coefficients (G, H). The last column in Table 2 shows the Euler coordinates when the origin is set at X(2) or x_e'(n) = x_e(n) - x_e(2). Then, all x_e'(n) except X(30) are constants within the unit (E-8F), This means that the distance between every two centers is a real number within the unit (E-8F).

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Table 2 Euler coordinates ($x_e(n)$, 0) for X(n) on the Euler line. (G, H) are S-coefficients and $x_e(n)' = x_e(n) - x_e(2)$ corresponding to the Euler coordinates when the origin is set at X(2).

X(n)	(G(n), H(n))	$x_e(n)$	$x_e(n)'$/(E-8F)
X(2)	(1, 0)	(E+F)/3	0
X(3)	(1, -1)	(E-2F)/2	1/6
X(4)	(0, 1)	3F	-1/3
X(5)	(1, 1)	(E+4F)/4	-1/12
X(20)	(1, -2)	(E-5F)	2/3
X(30)	(1, -3)	(E-8F)/3	-3F/(E-8F)
X(140)	(3, -1)	3E/8	1/24
X(376)	(2, -3)	(2E-7F)/3	1/3
X(381)	(1, 3)	(E+10F)/6	-1/6
X(382)	(-1, 5)	-(E-14F)/2	-5/6
X(546)	(1, 5)	(E+16F)/8	-5/24
X(547)	(7, 3)	(7E+16F)/24	-1/24
X(548)	(5, -7)	(5E-16F)/8	7/24
X(549)	(5, -3)	(5E-4F)/12	1/12
X(550)	(3, -5)	3(E-4F)/4	5/12
X(631)	(2, -1)	(2E-F)/5	1/15
X(632)	(7, -1)	(7E+4F)/20	1/60
X(1656)	(3, 1)	3(E+2F)/10	-1/30
X(1657)	(3, -7)	3(E-6F)/2	7/6
X(3090)	(2, 1)	(2E+5F)/7	-1/21
X(3091)	(1, 2)	(E+7F)/5	-2/15
X(3146)	(-1, 4)	-(E-11F)	-4/3
X(3522)	(3, -4)	(3E-9F)/5	4/15
X(3523)	(3, -2)	3(E-F)/7	2/21
X(3524)	(4, -3)	(4E-5F)/9	1/9
X(3525)	(4, -1)	(4E+F)/11	1/33
X(3526)	(5, -1)	(5E+2F)/14	1/42

X(3528)	(4, -5)	(4E-11F)/7	5/21
X(3529)	(2, -5)	(2E-13F)	5/3
X(3530)	(7, -5)	(7E-8F)/16	5/48
X(3533)	(6, -1)	(6E+3F)/17	1/51
X(3534)	(5, -9)	(5E-22F)/6	1/2
X(3543)	(-1, 6)	-(E-17F)/3	-2/3
X(3544)	(4, 5)	(4E+19F)/17	-5/51
X(3545)	(2, 3)	(2E+11F)/9	-1/9
X(3627)	(-1, 7)	-(E-20F)/4	-7/12
X(3628)	(5, 1)	(5E+8F)/16	-1/48
X(3830)	(-1, 9)	-(E-26F)/6	-1/2
X(3832)	(1, 4)	(E+13F)/7	-4/21
X(3839)	(1, 6)	(E+19F)/9	-2/9
X(3843)	(1, 7)	(E+22F)/10	-7/30
X(3845)	(1, 9)	(E+28F)/12	-1/4
X(3850)	(3, 7)	3(E+8F)/16	-7/48
X(3851)	(3, 5)	(3E+18F)/14	-5/42
X(3853)	(-1, 11)	-(E-32F)/8	-11/24
X(3854)	(3, 8)	3(E+9F)/17	-8/51
X(3855)	(2, 5)	(2E+17F)/11	-5/33
X(3856)	(5, 17)	(5E+56F)/32	-17/96
X(3857)	(5, 13)	(5E+44F)/28	-13/84
X(3858)	(3, 11)	3(E+12F)/20	-11/60
X(3859)	(7, 19)	(7E+64F)/40	-19/120
X(3860)	(7, 27)	(7E+88F)/48	-9/48
X(3861)	(1, 13)	(E+40F)/16	-13/48
X(5054)	(7, -3)	(7E-2F)/18	1/18
X(5055)	(5, 3)	(5E+14F)/18	-1/18
X(5056)	(3, 2)	3(E+3F)/11	-2/33
X(5059)	(3, -8)	3(E-7F)	8/3
X(5066)	(5, 9)	(5E+32F)/24	-1/8
X(5067)	(4, 1)	(4E+7F)/13	-1/39
X(5068)	(3, 4)	3(E+5F)/13	-4/39
X(5070)	(7, 1)	(7E+10F)/22	-1/66
X(5071)	(4, 3)	(4E+13F)/15	-1/15

X(5072)	(5, 7)	(5E+26F)/22	-7/66
X(5073)	(-3, 11)	-3(E-10F)/2	-11/6
X(5076)	(-1, 13)	-(E-38F)/10	-13/30
X(5079)	(7, 5)	(7E+22F)/26	-5/78
X(7486)	(5, 2)	(5E+11F)/17	-2/51
X(8703)	(7, -9)	(7E-20F)/12	1/4
X(10109)	(13, 9)	(13E+40F)/48	-1/16
X(10124)	(17, -3)	(17E+8F)/48	1/48
X(10299)	(6, -5)	(6E-9F)/13	5/39
X(10303)	(5, -2)	(5E-F)/13	2/39
X(10304)	(5, -6)	(5E-13F)/9	2/9
X(11001)	(4, -9)	(4E-23F)/3	1
X(11539)	(13, -3)	(13E+4F)/36	1/36
X(11540)	(35, -9)	(35E+8F)/96	1/32
X(11541)	(-4, 13)	-(4E-35F)	-13/3
X(11737)	(11, 15)	(11E+56F)/48	-5/48
X(11812)	(19, -9)	(19E-8F)/48	1/16
X(12100)	(11, -9)	(11E-16F)/24	1/8
X(12101)	(-1, 27)	-(E-80F)/24	-3/8
X(12102)	(-1, 19)	-(E-56F)/16	-19/48
X(12103)	(7, -13)	(7E-32F)/8	13/24
X(12108)	(13, -7)	(13E-8F)/32	7/96
X(12811)	(7, 11)	(7E+40F)/32	-11/96
X(12812)	(11, 7)	(11E+32F)/40	-7/120
X(14093)	(17, -21)	(17E-46F)/30	7/30
X(14269)	(1, 15)	(E+46F)/18	-5/18
X(14869)	(11, -5)	(11E-4F)/28	5/84
X(14890)	(55, -21)	(55E-8F)/144	7/144
X(14891)	(23, -21)	(23E-40F)/48	7/48
X(14892)	(17, 21)	(17E+80F)/72	-7/72
X(14893)	(1, 21)	(E+64F)/24	-7/24
X(15022)	(5, 4)	(5E+17F)/19	-4/57
X(15640)	(-5, 18)	-(5E-49F)/3	-2
X(15681)	(7, -15)	(7E-38F)/6	5/6
X(15682)	(-2, 9)	-(2E-25F)/3	-1

X(15683)	(5, -12)	(5E-31F)/3	4/3
X(15684)	(-5, 21)	-(5E-58F)/6	-7/6
X(15685)	(11, -27)	(11E-70F)/6	3/2
X(15686)	(11, -21)	(11E-52F)/12	7/12
X(15687)	(-1, 15)	-(E-44F)/12	-5/12
X(15688)	(11, -15)	(11E-34F)/18	5/18
X(15689)	(13, -21)	(13E-50F)/18	7/18
X(15690)	(17, -27)	(17E-64F)/24	3/8
X(15691)	(19, -33)	(19E-80F)/24	11/24
X(15692)	(7, -6)	(7E-11F)/15	2/15
X(15693)	(13, -9)	(13E-14F)/30	1/10
X(15694)	(11, -3)	(11E+2F)/30	1/30
X(15695)	(19, -27)	(19E-62F)/30	3/10
X(15696)	(7, -11)	(7E-26F)/10	11/30
X(15697)	(11, -18)	(11E-43F)/15	2/5
X(15698)	(10, -9)	(10E-17F)/21	1/7
X(15699)	(11, 3)	(11E+20F)/36	-1/36
X(15700)	(19, -15)	(19E-26F)/42	5/42
X(15701)	(17, -9)	(17E-10F)/42	1/14
X(15702)	(8, -3)	(8E-F)/21	1/21
X(15703)	(13, 3)	(13E+22F)/42	-1/42
X(15704)	(5, -11)	(5E-28F)/4	11/12
X(15705)	(13, -12)	(13E-23F)/27	4/27
X(15706)	(25, -21)	(25E-38F)/54	7/54
X(15707)	(23, -15)	(23E-22F)/54	5/54
X(15708)	(11, -6)	(11E-7F)/27	2/27
X(15709)	(10, -3)	(10E+F)/27	1/27
X(15710)	(14, -15)	(14E-31F)/27	5/27
X(15711)	(29, -27)	(29E-52F)/60	3/20
X(15712)	(9, -7)	(9E-12F)/20	7/60
X(15713)	(23, -9)	(23E-4F)/60	1/20
X(15714)	(31, -33)	(31E-68F)/60	11/60
X(15715)	(16, -15)	(16E-29F)/33	5/33
X(15716)	(31, -27)	(31E-50F)/66	3/22
X(15717)	(5, -4)	(5E-7F)/11	4/33

X(15718)	(29, -21)	(29E-34F)/66	7/66
X(15719)	(14, -9)	(14E-13F)/33	1/11
X(15720)	(9, -5)	3(3E-2F)/22	5/66
X(15721)	(13, -6)	(13E-5F)/33	2/33
X(15722)	(43, -27)	(43E-38F)/102	3/34
X(15723)	(23, -3)	(23E+14F)/66	1/66
X(15759)	(25, -27)	(25E-56F)/48	3/16
X(16239)	(11, -1)	(11E+8F)/32	1/96
X(17504)	(17, -15)	(17E-28F)/36	5/36
X(17538)	(4, -7)	(4E-17F)/5	7/15
X(17578)	(-1, 8)	-(E-23F)/5	-8/15
X(17800)	(5, -13)	(5E-34F)/2	13/6
X(19708)	(8, -9)	(8E-19F)/15	1/5
X(19709)	(7, 9)	(7E+34F)/30	-1/10
X(19710)	(13, -27)	(13E-68F)/12	3/4
X(19711)	(37, -27)	(37E-44F)/84	3/28
X(21734)	(7, -8)	(7E-17F)/13	8/39
X(21735)	(6, -7)	(6E-15F)/11	7/33
X(23046)	(5, 21)	(5E+68F)/36	-7/36
X(33699)	(-5,27)	-(5E-76F)/12	-3/4
X(33703)	(-2, 7)	-(2E-19F)	-7/3
X(33923)	(9, -11)	(9E-24F)/16	11/48
X(34200)	(13, -15)	(13E-32F)/24	5/24
X(35018)	(9, 5)	(9E+24F)/32	-5/96
X(35381)	(119, -27)	(119E+38F)/330	3/110
X(35382)	(101, 87)	(101E+362F)/390	-29/390
X(35384)	(-59, 207)	-(59E-562F)/30	-23/10
X(35400)	(-17, 57)	-(17E-154F)/6	-19/6
X(35401)	(-7, 87)	-(7E-254F)/66	-29/66
X(35402)	(-11, 111)	-(11E-322F)/78	-37/78
X(35403)	(-1, 33)	-(E-98F)/30	-11/30
X(35404)	(-7, 33)	-(7E-92F)/12	-11/12
X(35405)	(-39, 139)	-(39E-378F)/22	-139/66
X(35406)	(-47, 167)	-(47E-454F)/26	-167/78
X(35407)	(-19, 67)	-(19E-182F)/10	-67/30

X(35408)	(-59, 201)	-(59E-544F)/24	-67/24
X(35409)	(-14, 51)	-(14E-139F)/9	-17/9
X(35410)	(191, -507)	(191E-1330F)/66	169/66
X(35411)	(223, -591)	(223E-1550F)/78	197/78
X(35412)	(199, -519)	(199E-1358F)/78	173/78
X(35413)	(205, -519)	(205E-1352F)/96	173/96
X(35414)	(49, -132)	(49E-347F)/15	44/15
X(35415)	(263, -503)	(263E-1246F)/286	503/858
X(35416)	(-41, 233)	-(41E-658F)/110	-233/330
X(35417)	(119, 699)	(119E+2216F)/1056	-233/1056
X(35418)	(61, -84)	(61E-191F)/99	28/99
X(35419)	(-79, 367)	-(79E-1022F)/130	-367/390
X(35420)	(101, 1101)	(101E+3404F)/1404	-367/1404
X(35421)	(203, -297)	(203E-688F)/312	33/104
X(35434)	(-7, 51)	-(7E-146F)/30	-17/30
X(35435)	(205, -537)	(205E-1406F)/78	179/78
X(38071)	(7, 15)	(7E+52F)/36	-5/36
X(38335)	(-1, 21)	-(E-62F)/18	-7/18
X(41982)	(41, -51)	(41E-112F)/72	17/72
X(41983)	(31, -21)	(31E-32F)/72	7/72
X(41984)	(49, -3)	(49E+40F)/144	1/144
X(41985)	(47, 3)	(47E+56F)/144	-1/144
X(41986)	(73, 69)	(73E+280F)/288	-23/288
X(41987)	(7, 51)	(7E+160F)/72	-17/72
X(41988)	(1, 93)	(E+280F)/96	-31/96
X(41989)	(19, 23)	(19E+88F)/80	-23/240
X(41990)	(31, 63)	(31E+220F)/156	-7/52
X(41991)	(7, 23)	(7E+76F)/44	-23/132
X(41992)	(31, -1)	(31E+28F)/92	1/276

From Table 2, many combos, for example, **3X(2)=2X(3)+X(4) = -X(3) +4X(5)** etc. can be easily obtained. In particular, the midpoints of (X(3), X(381)), (X(4), X(376)), (X(5), X(549)), (X(20), X(3543)), (X(140), X(547)), (X(382), X(15681)), (X(546), X(34200)), (X(547), X(15687)), (X(631), X(5071)), (X(1656), X(15694)), (X(1657), X(15684)), (X(548), X(14893)), (X(548), X(14893)), (X(550), X(15687)), (X(631), X(5071)), (X(3090), X(15702)), (X(3091), X(15692)), (X(3146), X(15683)), (X(3524), X(3545)), (X(3526), X(15703)), (X(3530), X(11737)), (X(3534), X(3830)), (X(3627), X(15686)), (X(3628), X(10124)), (X(3839), X(10304)), (X(3843), X(14093)), (X(3845), X(8703)), (X(3850), X(14891)), (X(3851), X(15700)), (X(3853), X(15691)), (X(3855), X(15715)), (X(3858), X(15714)), (X(5054), X(5055)), (X(5056), X(15721)), (X(5066), X(12100)), (X(10109), X(11812)), (X(11001), X(15682)), (X(11539), X(15699)), (X(12101), X(15690)), (X(14269), X(15688)), (X(14892), X(41983)), (X(15689), X(38335)), (X(15693), X(19709)), (X(15696), X(35403)), (X(15704), X(35404)), (X(17504), X(38071)), (X(19710), X(33699)), (X(41982), X(41987)) and (X(41984), X(41985)) are X(2).

Table 3 shows the Euler coordinates (x_e, 0) for the triangle center X(n) on the Euler line whose normalization constant is proportional to (E-8F)S^2.

Table 3 S-coefficients (G, H) and Euler coordinates (x_e(n), 0) for X(n) whose normalization constant N are proportional to (E-8F)S^2.

X(n)	(G, H)	x_e(n)
X(23)	(-E-4F, 4E+4F)	-(E+F)
X(186)	(-4F, E+4F)	-F
X(403)	(-2F, E-2F)	F
X(468)	(-3F, E+F)	0
X(858)	(E-2F, -2E-2F)	(E+F)
X(2070)	(-E-8F, 5E+8F)/2	-(E+2F)/2
X(2071)	(E-4F, -2E+4F)	(E-F)
X(2072)	(E-4F, -E-4F)/2	(E+2F)/2
X(3153)	(E, -2E-8F)	(E+3F)
X(5159)	(E-5F, -E-F)/2	(E+F)/2
X(5189)	(3E, -8E-8F)	3(E+F)
X(5899)	(-3E-8F, 11E+8F)/2	-(3E+2F)/2
X(7426)	(-E-10F, 6E+6F)/3	-(E+F)/3
X(7464)	(2E-4F,-5E+4F)	(2E-F)
X(7574)	(3E, -7E-16F)/2	3(E+F)/2
X(7575)	(-E-16F, 7E+16F)/4	-(E+4F)/4
X(10096)	(-E-24F, 11E+8F)/8	-E/8

X(10151)	(-F, E-5F)	2(E-8F)
X(10257)	(E-6F, -E+2F)/2	E/2
X(10295)	(-6F, E+10F)	-3F
X(10296)	(E+4F, -2E-20F)	(E+7F)
X(10297)	(E-2F, -E-10F)/2	(E+4F)/2
X(10989)	(5E-4F, -12E-12F)/3	5(E+F)/3
X(11558)	(-3E-8F, 17E-40F)/8	-(3E-16F)/8
X(11563)	(-E-8F, 7E-8F)/4	-(E-4F)/4
X(11799)	(-E-4F, 5E-4F)/2	-(E-2F)/2
X(12105)	(-5E-32F, 23E+32F)/8	-(5E+8F)/8
X(13473)	(F, E-11F)	4F
X(13619)	(-8F, E+16F)	-5F
X(13350)	(3E-40F,7E-8F)/16	(3E+8F)/16
X(15122)	(3E-12F, -5E+4F)/4	3E/4
X(15646)	(E-16F, E+16F)/4	(E-4F)/4
X(16386)	(E-6F, -2E+10F)	(E-3F)
X(16532)	(E-40F, 9E+24F)/12	(E-4F)/12
X(16619)	(-3E-12F, 13E+4F)/4	-3F/4
X(16976)	(E-7F, -E+5F)/2	(E-F)/2
X(18323)	(E+4F, -E-28F)/2	(E+10F)/2
X(18325)	(-3E, 11E-16F)/2	-3(E-2F)/2
X(18403)	(E, -E-16F)/2	(E+6F)/2
X(18571)	(E-32F, 5E+32F)/8	(E-8F)/8
X(18572)	(3E, -5E-32F)./4	3(E+4F)/4
X(18579)	(E-44F, 9E+36F)/12	(E-8F)/12
X(18859)	(3E-8F, -7E+8F)/2.	(3E-2F)/2
X(20063)	(-5E-8F, 16E+16F).	-5(E+F)
X(22248)	(-5E-152F, 55E+136F)/40	-(5E+32F)/40
X(22249)	(E-56F, 13E+40F)/16.	(E-8F)/16
X(23323)	(E-4F, E-20F)/4	(E+8F)/4
X(25338)	(-3E-24F, 17E+8F)/8	-3E/8
X(31726)	(-E, 5E-16F)/2	-(E-6F)/2
X(34152)	(3E-16F, -5E+16F)/4	(3E-4F)/4
X(35001)	(7E-8F, -19E+8F)/2	(7E-2F)/2
X(35452)	(5E-8F, -13E+8F)/2	(5E-2F)/2

X(35489)	(-16F, 3E+24F)/3	-7F/3
X(37760)	(-E-16F, 8E+8F)/5	-(E+F)/5
X(37897)	(-E-7F,5E+5F)/2	-(E+F)/2
X(37899)	(-2E-5F, 7E+7F)	-2(E+F)
X(37900)	(-3E-6F, 10E+10F)	-3(E+F)
X(37901)	(-7E-16F, 24E+24F)/3	-7(E+F)/3
X(37904)	(-2E-11F, 9E+9F)/3	-2(E+F)/3
X(37907)	(-E-28F, 12E+12F)/9	-(E+F)/9
X(37909)	(-5E-32F, 24E+24F)/9	-5(E+F)/9
X(37910)	(-3E-9F, 11E+11F)/2	-3(E+F)/2
X(37911)	(E-11F, E+F)/4	(E+F)/4
X(37922)	(-E-24F, 9E+24F)/6	-(E+6F)/6
X(37923)	(-7E-40F, 31E+40F)/10	-(7E+10F)/10
X(37924)	(-5E-8F, 17E+8F)/2	-(5E+2F)/2
X(37925)	(-2E-4F, 7E+4F)	-(2E+F)
X(37931)	(-5F, E+7F)	-2F
X(37934)	(-9F, 2E+11F)/2	-3F/2
X(37935)	(-7F, 2E+5F)/2	-F/2
X(37936)	(-9E-48F, 39E+48F)/12	-(3E+4F)/4
X(37938)	(3E-8F, -5E-8F)/4	(3E+4F)/4
X(37939)	(-2E-12F, 9E+12F)/3	-(2E+3F)/3
X(37940)	(-E-12F, 6E+12F)/3	-(E+3F)/3
X(37941)	(E-12F, 12F)/3	(E-3F)/3
X(37942)	(-5F, 2E-F)/2	F/2
X(37943)	(8F, -3E)/3	F/3
X(37944)	(3E-4F, -8E+4F)	(3E-F)
X(37945)	(-3E-4F, 10E+4F)	-(3E+F)
X(37946)	(-4E-4F, 13E+4F)	-(4E+F)
X(37947)	(-5E-16F, 19E+16F)/4	-(5E+4F)/4
X(37948)	(2E-12F, -3E+12F)/3	(2E+3F)/3
X(37949)	(-7E-8F, 23E+8F)/2	-(7E+2F)/2
X(37950)	(5E-16F, -11E+16F)/4	(5E+4F)/4
X(37952)	(E-20F, 2E+20F)/5	(E-5F)/5
X(37953)	(-2E-20F, 11E+20F)/5	-(2E+5F)/5
X(37955)	(E-24F, 3E+24F)/6	(E-6F)/6

X(37956)	(-5E-24F, 21E+24F)/6	-(5E+6F)/6
X(37957)	(-E-28F, 10E+28F)/7	-(E+7F)/7
X(37958)	(-E-40F, 13E+40F)/10	-(E+10F)/10
X(37967)	(-7E-16F, 25E+16F)/4	-(7E+4F)/4
X(37968)	(3E-32F, -E+32F)/8	(3E-8F)/8
X(37971)	(-E-6F, 5E+2F)/2	-E/2
X(37984)	(-3F, 2E-7F)/2	3F/2
X(43893)	(-3E-8F,13F-8F)/4	-(3E-4F)/4

From Table 2 and Table 3, the midpoints of (X(2), X(7426)), (X(3), X(11799)), (X(4), X(10295)), (X(5), X(7575)), (X(140), X(25338)), (X(23), X(858)), (X(186), X(403)), (X(2070), X(2072)), (X(5189), X(37897)), (X(7574), X(37910)), (X(10256), X(37971)), (X(11563), X(15646)), (X(34152), X(43893)), (X(37934), X(37984)), (X(37935), X(37942)), (X(37936), X(37938)), (X((37939), X(37948)) and (X(37947), X(37950)) are X(468). In addition, many combos can be obtained, for example, **X(23)+3X(2) = X(23)+2X(5159) = 3X(23)+X(5189) = X(468),X(23)+X(186) =2X(2070)**, etc..

Table 4 shows the Euler coordinates (x_e, y_e) for X(n) not on the Euler line whose normalization constant is proportional to (E-8F)S^2..

Table 4 Euler coordinates (x_e(n), y_e(n)) of X(n) not on the Euler line, whose N are proportional to (E-8F)S^2, $y_e(n)^* = y_e(n)(E-8F)/\$S_A^2(S_B-S_C)\$$

X(n)	Barycentrics	x_e(n)(E-8F)	$y_e(n)^*$
X(74)	$2(E-2F)S^2-3(E-2F)S_BS_C-2S_AS^2$	$-3(5E-4F)F+4S^2$	-2
X(110)	$-(E+4F)S^2+2(E+F)S_BS_C+2S_AS^2$	$(E^2+5EF+4F^2)-4S^2$	2
X(113)	$\{-(E+4F)S^2+3(E-2F)S_BS_C$ $+2S_AS^2\}/2$	$\{(E^2+8EF-20F^2)-4S^2\}/2$	1
X(125)	$(E-2F)S^2-(E+F)S_BS_C-S_AS^2$	$-6(E+F)F+2S^2$	-1
X(146)	$-3ES^2+6(E-2F)S_BS_C+4S_AS^2$	$(E^2+23EF-32F^2)-8S^2$	4
X(265)	$\{3ES^2 -3(E+4F)-4S_AS^2\}//2$	$\{-(E^2+14EF+40F^2)+8S^2\}/2$	-2
X(323)	$-(E+4F)S^2+4S_AS^2$	$3(E+F)E-8S^2$	4
X(399)	$\{-(5E+8F)S^2+9ES_BS_C+8S_AS^2\}/2$	$\{3(E+10F)F-16S^2\}/2$	4
X(468)	$-3FS^2+(E+F)S_BS_C$	0	0
X(1495)	$-(E+4F)S^2+3(E+F)+S_AS^2$	$6(E+F)F-2S^2$	1

X(1511)	$\{-(E+16F)S^2+3(E+4F)S_BS_C +4S_AS^2\}/4$	$\{3(E^2+8F^2)-8S^2\}/4$	1
X(1514)	$-(E+F)S^2+3(E-2F)S_BS_C+S_AS^2$	$9(E-2F)F-2S^2$	1
X(1531)	$-9FS_BS_C+ S_AS^2$	$(E^2+2EF-26F^2)-2S^2$	1
X(1533)	$-2(E+F)S^2+3(2E-F)+S_AS^2$	$-(E^2-16EF+10F^2)-2S^2$	1
X(1539)	$\{-3ES^2+9(E-4F)S_BS_C+4S_AS^2\}/4$	$\{(E^2+32EF-104F^2)-8S^2\}/4$	1
X(1568)	$-2FS^2-3FS_BS_C+S_AS^2$	$(E^2-10F^2)-2S^2$	1
X(3292)	$-(E+4F)S^2+(E+F)S_BS_C+3S_AS^2$	$2(E^2+2EF+F^2)-6S^2$	3
X(3448)	$3ES^2-4(E+F)-4S_AS^2$	$-(E^2+17EF+16F^2)+8S^2$	4
X(3580)	$(E-2F)S^2-2S_AS^2$	$-(E^2+5EF+4F^2)+4S^2$	-2
X(3581)	$\{(E-8F)S^2+3(E+4F)-4S_AS^2\}/2$	$\{-3(E^2+2EF-8F^2)+8S^2\}/2$	-2
X(5609)	$\{-(7E+16F)S^2+(13E+4F)S_BS_C +12S_AS^2\}/4$	$\{(5E^2+40EF+8F^2)-24S^2\}/4$	3
X(5642)	$\{-(E+10F)S^2+3(E+F)S_BS_C +3S_AS^2\}/3$	$\{(2E^2+4EF+2F^2)-6S^2\}/3$	1
X(5655)	$\{-(7E+16F)S^2+3(5E-4F)S_BS_C +12S_AS^2\}/6$	$\{(5E^2+46EF-40F^2)-24S^2\}/6$	2
X(5972)	$\{-6FS^2+(E+F)S_BS_C+ S_AS^2\}/2$	$\{(E^2-EF-2F^2)-2S^2\}/2$	1/2
X(6053)	$\{-3(E+2F)S^2+3(2E-F)S_BS_C +5S_AS^2\}/2$	$\{(2E^2+19EF-10F^2)-10S^2\}/2$	5/2
X(6699)	$\{3(E-4F)S^2-3(E-2F)S_BS_C-2S_AS^2\}/4$	$\{(E^2-22EF+4F^2)+4S^2\}/4$	-1/2
X(6723)	$\{2(E-5F)S^2-(E+F)S_BS_C -S_AS^2\}/4$	$\{(E^2-13EF-14F^2)+2S^2\}/4$	-1/4
X(7687)	$\{(E-2F)S^2-9FS_BS_C -S_AS^2\}/2$	$\{-3(E+10F)F+2S^2\}/2$	-1/2
X(7728)	$\{-3ES^2+(7E-20F)S_BS_C+4S_AS^2\}/2$	$\{(E^2+26EF-56F^2)-8S^2\}/2$	2
X(9140)	$\{(5E-4F)S^2-6(E+F)S_BS_C -6S_AS^2\}/3$	$\{-(E^2+29EF+28F^2)+12S^2\}/3$	-2
X(9143)	$\{-(7E+16F)S^2+12(E+F)S_BS_C + 12S_AS^2\}/3$	$\{(5E^2+37EF+32F^2)-24S^2\}/3$	4
X(10113)	$\{3ES^2-(E+28F)S_BS_C -4S_AS^2\}/4$	$-\{(E^2+8EF+88F^2)-8S^2\}/4$	-1
X(10264)	$\{(7E-8F)S^2-9ES_BS_C-8S_AS^2\}/4$	$\{-(E^2+44EF+16F^2)+16S^2\}/4$	-2
X(10272)	$\{-3(E+8F)S^2+9ES_BS_C+8S_AS^2\}/8$	$\{(5E^2+16EF-16F^2)-16S^2\}/8$	1
X(10540)	$\{-(3E+8F)S^2+(7E+4F)S_BS_C +4S_AS^2\}/2$	$\{(E^2+18EF+8F^2)-8S^2\}/2$	2
X(10564)	$\{(E-8F)S^2-3(E-2F)S_BS_C+2S_AS^2\}/2$	$\{3(E^2-4EF+4F^2)-4S^2\}/2$	1
X(10620)	$\{(7E-8F)S^2-(11E-16F)S_BS_C -8S_AS^2\}/2$	$\{-(E^2+50EF-32F^2)+16S^2\}/2$	-4

X(10706)	$\{-4(E+F)S^2+9(E-2F)S_BS_C$ $+6S_AS^2\}/3$	$\{(2E^2+31EF-52F^2)-12S^2\}/3$	2
X(10721)	$-2(E-2F)S^2+(5E-22F)S_BS_C\ 2S_AS^2$	$3(7E-20F)F-4S^2$	2
X(10733)	$(E+4F)S^2-18FS_BS_C-2S_AS^2$	$-(E^2-EF+52F^2)+4S^2$	-2
X(10990)	$3(E-2F)S^2-(5E\ -13F)S_BS_C-3S_AS^2$	$-6\{(4E-5F)F-S^2\}$	-3
X(11064)	$-3FS^2+S_AS^2$	$(E^2-EF-2F^2)-2S^2$	1
X(11693)	$\{-(5E+68F)S^2+(15E+42F)S_BS_C$ $+18S_AS^2\}/18$	$\{(13E^2+8EF+76F^2)-36S^2\}$ $/18$	1
X(11694)	$\{-7(E+88F)S^2+3(7E+16F)S_BS_C$ $+24S_AS^2\}.24$	$\{(17E^2+16EF+80F^2)-48S^2\}$ $/24$	1
X(11801)	$\{(7E-8F)S^2-(5E+32F)S_BS_C$ $-8S_AS^2\}/8$	$\{-(E^2+32EF+112F^2)+16S^2\}$ $/8$	-1
X(12041)	$\{(5E-16F)S^2$ –$(7E-20F)S_BS_C$ $-4S_AS^2\}/4$	$\{(E^2-40EF+40F^2)+8S^2\}/4$	-1
X(12112)	$-4(E+F)S^2+\ 9ES_BS_C+4S_AS^2$	$27EF-8S^2$	4
X(12121)	$\{-(E+16F)S^2+(\ E+28F)S_BS_C$ $+4S_AS^2\}/2$	$\{3(E^2-2EF+24F^2)-8S^2\}/2$	2
X(12244)	$4(E-2F)S^2-(7E-20F)S_BS_C-4S_AS^2$	$-3(11E-16F)F+8S^2$	-4
X(12295)	$\{(E+4F)S^2+(E-26F)S_BS_C-2S_AS^2\}/2$	$\{-(E^2-4EF+76F^2)+4S^2\}/2$	-1
X(12308)	$-(11E+8F)S^2+(19E-8F)S_BS_C$ $+16S_AS^2$	$(5E^2+70EF-16F^2)-32S^2$	8
X(12317)	$6ES^2-9ES_BS_C-8S_AS^2$	$-(2E^2+37EF+8F^2)+16S^2$	-8
X(12383)	$-2(E+4F)S^2+3(E+4F)S_BS_C+4S_AS^2$	$(2E^2+7EF+32F^2)-8S^2$	4
X(12900)	$\{(E-20F)S^2+3(E-2F)S_BS_C$ $+2S_AS^2\}/8$	$\{\{3(E^2-2EF-12F^2)-4S^2\}$ $/8$	1/4
X(12902)	$\{(5E+8F)S^2-(5E+32F)S_BS_C$ $-8S_AS^2\}/2$	$\{-3(E^2+6EF+32F^2)+16S^2\}$ $/2$	-4
X(13202)	$-(E-2F)S^2+3(E-5F)S_BS_C+S_AS^2$	$6(2E-7F)F-2S^2$	1
X(13392)	$\{-(5E+56F)S^2+3(5E+8F)S_BS_C$ $+16S_AS^2\}/16$	$\{(11E^2+16EF+32F^2)-32S^2\}$ $/16$	1
X(13393)	$\{3(13E-8F)S^2-(53E+8F)S_BS_C$ $-48S_AS^2\}/16$	$-3\{(3E^2+80EF+32F^2)-32S^2\}$ $/16$	3
X(13399)	$(3E-2F)S^2-(5E-F)S_BS_C-3S_AS^2$	$-2(10E+F)F+6S^2$	-3
X(13455)	$(3E-4F)S^2-6(E-F)S_BS_C-2S_AS^2$	$(E^2-23EF+12F^2)+4S^2$	-2
X(13851)	$ES^2\ -(E+7F)S_BS_C-S_AS^2$	$-2\{(2E+11F)-S^2\}$	1

X(13857)	$\{(E-8F)S^2-3(E+F)S_BS_C+3S_AS^2\}/3$	$\{2(2E^2-5EF-7F^2)-6S^2\}/3$	1
X(14094)	$-4(E+F)S^2+(7E-2F)S_BS_C+6S_AS^2$	$(2E^2+25EF-4F^2)-12S^2$	6
X(14156)	$\{(E-12F)S^2-(E-2F)S_BS_C+2S_AS^2\}/4$	$\{(3E^2-10EF-4F^2)-4S^2\}/4$	1/2
X(14157)	$-2(E+2F)S^2+(5E+2F)S_BS_C+2S_AS^2$	$(13E+4F)F-4S^2$	2
X(14643)	$\{-(E+16F)S^2+(5E-4F)S_BS_C$ $+4S_AS^2\}/6$	$\{3(E^2+2EF-8F^2)-8S^2\}/6$	2/3
X(14644)	$\{2(E-2F)S^2-(E+10F)S_BS_C-2S_AS^2\}/3$	$\{-9(E+4F)F+4S^2\}/3$	-2/3
X(14677)	$\{3(3E-8F)S^2-3(5E-16F)S_BS_C$ $-8S_AS^2\}/4$	$\{(E^2-76EF+112F^2)+16S^2\}/4$	-2
X(14683)	$-(5E+8F)S^2+8(E+F)S_BS_C+8S_AS^2$	$3(E^2+9EF+F^2)-16S^2$	8
X(15020)	$\{-(E+28F)S^2+2(2E+11F)S_BS_C$ $+6S_AS^2\}/7$	$\{5E^2-5EF+44F^2-12S^2\}/7$	6/7
X(15021)	$\{(7E-20F)S^2-2(5E-13F)S_BS_C$ $-6S_AS^2\}/5$	$\{(E^2-55EF+52F^2)+12S^2\}/5$	-6/5
X(15023)	$\{(5E-76F)S^2-2(E-35F)S_BS_C$ $+6S_AS^2\}/19$	$\{(11E^2-65EF-140F^2)-12S^2\}$ $/19$	6/19
X(15025)	$\{(7E-20F)S^2-2(2E+11F)S_BS_C$ $-6S_AS^2\}/17$	$\{(E^2-37EF-92F^2)-12S^2\}/17$	-6/17
X(15027)	$\{(11E-16F)S^2-(11E+20F)S_BS_C$ $-12S_AS^2)/10$	$\{-(E^2+62EF+88F^2)+24S^2\}$ $/10$	-6/5
X(15029)	$\{-(E+28F)S^2+2(5E-13F)S_BS_C$ $+6S_AS^2\}/13$	$\{(5E^2+13EF-100F^2)-12S^2\}$ $/13$	6/13
X(15034)	$\{-2(E+10F)S^2+(5E+14F)S_BS_C$ $+6S_AS^2\}/5$	$\{(4E^2+5EF+28F^2)-12S^2\}/5$	6/5
X(15035)	$\{-12FS^2+(E+10F)S_BS_C+2S_AS^2\}/3$	$\{(2E^2-5EF+20F^2)-4S^2\}/3$	2/3
X(15036)	$\{2(E-14F)S^2-(E-26F)S_BS_C$ $+2S_AS^2)/7$	$\{(4E^2-25EF+52F^2)-4S^2\}/7$	2/7
X(15039)	$\{-(11E+56F)S^2+(23E+32F)S_BS_C$ $+24S_AS^2\}/14$	$\{13E^2+50EF+64F^2\}-48S^2\}$ $/14$	12/7
X(15040)	$\{-(E+40F)S^2+(5E+32F)S_BS_C$ $+8S_AS^2\}/10$	$\{7E^2-10EF+64F^2\}-16S^2\}/10$	3/5
X(15041)	$\{3(3E-8F)S^2-(13E-32F)S_BS_C$ $-8S_AS^2\}/6$	$\{(E^2-70EF+64F^2)+16S^2\}/6$	4/3
X(15042)	$\{(7E-104F)S^2 -3(E-32F)S_BS_C$ $+8S_AS^2\}/26$	$\{3(5E^2-30EF+64F^2\}-16S^2$ $/26$	4/13

X(15044)	$\{(5E-4F)S^2 -2(E+19F)S_BS_C$ $-6S_AS^2\}/7$	$\{-(E^2+17EF+124F^2)+12S^2\}$ $/7$	-6/7
X(15046)	$\{-(E+40F)S^2+(13E-32F)S_BS_C$ $+8S_AS^2\}/18$	$\{(7E^2+14EH-128F^2)-16S^2\}$ $/18$	4/9
X(15051)	$\{(E-20F)S^2+18FS_BS_C+ 2S_AS^2\}/5$	$\{3(E^2-5EF+12F^2)-4S^2\}/5$	2/5
X(15054)	$(5E-4F)S^2 -2(4E-5F)S_BS_C-6S_AS^2$	$-\{(E^2+35EF-20F^2)-12S^2\}$	-6
X(15055)	$3(E-4F)S^2 -2(2E-7F)S_BS_C-2S_AS^2$	$(E^2-25EF+28F^2)+4S^2$	-2
X(15057)	$\{(7E-20F)S^2 -2(4E-5F)S_BS_C$ $-6S_AS^2\}/7$	$\{(E^2-49EF+4F^2)+12S^2\}/7$	-6/7
X(15059)	$\{3(E-4F)S^2-2(E+F)S_BS_C-2S_AS^2\}/5$	$\{(E^2-19EF-20F^2)+4S^2\}/5$	-2/5
X(15061)	$\{(5E-16F)S^2-(5E-4F)S_BS_C$ $-4S_AS^2\}/6$	$\{(E^2-34EF-8F^2)+8S^2\}/6$	-2/3
X(15063)	$-2(E+F)S^2+(4E-5F)S_BS_C+ 3S_AS^2$	$(E^2+14EF-14F^2)-6S^2$	3
X(15081)	$\{4(E-2F)S^2-3(E+4F)S_BS_C-4S_AS^2\}/5$	$-\{3(7E+16F)F-8S^2\}/5$	-4/5
X(15088)	$\{(7E-32F)S^2-(E+28F)S_BS_C$ $-4S_AS^2\}/16$	$\{3(E^2-12EF-40F^2)+8S^2\}/16$	-1/4
X(15107)	$-(E+4F)S^2+6(E+F)S_BS_C -2S_AS^2$	$-3(E^2-3EF-4F^2)+4S^2$	-2
X(15360)	$\{(E-8F)S^2+ 6(E+F)S_BS_C -6S_AS^2\}/3$	$\{-(5E^2+EF-4F^2)+12S^2\}/3$	-2
X(15361)	$\{5(E-8F)S^2+9(E+4F)S_BS_C$ $-12S_AS^2\}/12$	$\{-(7E^2+32EF-56F^2)+24S^2\}$ $/12$	-1
X(15362)	$\{5(E-8F)S^2+3(5E-4F)S_BS_C,$ $-12S_AS^2\}/18$	$\{-(7E^2+14EF+88F^2)+24S^2\}$ $/18$	-2/3
X(15448)	$\{-(E+7F)S^2+4(E+F)S_BS_C+S_AS^2\}/2$	$3(E+F)F-S^2$	1/2
X(16003)	$\{(5E-4F)S^2-(7E-2F)S_BS_C-6S_AS^2\}/2$	$\{-(E^2+32EF+4F^2)+12S^2\}/2$	-3
X(16111)	$\{3(E-4F)S^2-(5E-22F)S_BS_C$ $-2S_AS^2\}/4$	$\{(E^2-28EF+52F^2)+4S^2\}/4$	-1
X(16163)	$-6FS^2+9FS_BS_C+S_AS^2$	$\{(E^2-4EF+22F^2)-2S^2\}$	1
X(16534)	$\{-3(E+4F)S^2+(7E-2F)S_BS_C$ $+6S_AS^2\}/4$	$3\{(E^2+6EF-4F^2)-4S^2\}/4$	3/2
X(20125)	$\{-4(E+4F)S^2+9ES_BS_C+8S_AS^2\}/5$	$\{(4E^2+23EF-8F^2)-16S^2\}/5$	8/5
X(20126)	$\{(11E-16F)S^2-3(5E-4F)S_BS_C$ $-12S_AS^2\}/6$	$-\{(E^2+74EF-8F^2)-24S^2\}/6$	-2
X(20127)	$\{(5E-16F)S^2-9(E-4F)S_BS_C$ $-4S_AS^2\}/2$	$\{(E^2-46EF+88F^2)+8S^2\}/2$	-2
X(20304)	$\{(5E-16F)S^2-3(E+4F)S_BS_C$	$\{(E^2-28EF-56F^2)+8S^2\}/8$	-1/2

	$-4S_AS^2\}/8$		
X(20396)	$\{(13E-32F)S^2-(11E+20F)S_BS_C,$ $-12S_AS^2\}/16$	$\{(E^2-76EF-104F^2)+24S^2\}$ $/16$	-3/4
X(20397)	$\{(11E-16F)S^2-(13E+4F)S_BS_C$ $-12S_AS^2\}/2$	$-\{(E^2+68EF+40F^2)-24S^2\}/2$	-6
X(20417)	$\{3(E-2F)S^2-(4E-5F)S_BS_C-3S_AS^2\}/2$	$-3\{(7E-2F)F-2S^2\}/2$	-3/2
X(21663)	$(E-4F)S^2 -(E-5F)S_BS_C -S_AS^2$	$-2\{(4E-5F)F-S^2\}$	-1
X(22115)	$\{-(E+8F)S^2+(E+4F)S_BS_C+4S_AS^2\}/2$	$\{(3E^2+2EF+8F^2)-8S^2\}/2$	2
X(22250)	$\{-3(7E+72F)S^2+(55E+136F)S_BS_C$ $+64S_AS^2\}/56$	$\{(43E^2+56EF+256F^2)$ $-128S^2\}/56$	8/7
X(22251)	$\{-3(E+24F)S^2+(13E+40F)S_BS_C$ $+16S_AS^2\}/20$	$\{(13E^2-4EF+64F^2)-32S^2\}$ $/20$	4/5
X(23061)	$-(E+4F^)S^2 -2(E+F)S_BS_C+6S_AS^2$	$(5E^2+EF-4F^2)-12S^2$	6
X(23236)	$\{-(7E+16F)S^2+(11E+20F)S_BS_C$ $+12S_AS^2\}/2$	$\{(5E^2+34EF+56F^2)-24S^2\}$ $/2$	6
X(23515)	$\{3(E-4F)S^2-(E+10F)S_BS_C$ $-2S_AS^2\}/6$	$\{(E^2-16EF-44F^2)+4S^2\}/6$	-1/3
X(24981)	$-3(E+2F)S^2+5(E+F)S_BS_C+5S_AS^2$	$2\{(E^2+8EF+7F^2)-5S^2\}$	5
X(25739)	$2ES^2 -3(E+2F)S_BS_C-2S_AS^2$	$-\{(11E+20F)F-4S^2\}$	-2
X(30714)	$-3(E+4F)S^2+(5E+14F)S_BS_C+6S_AS^2$	$3\{(E^2+4EF+12F^2)-4S^2\}$	6
X(32111)	$-2(E+F)S^2+(5E-4F)S_BS_C+2S_AS^2$	$3(5E-4F)F-4S^2$	2
X(32269)	$-3FS^2+2(E+F)S_BS_C-S_AS^2$	$-\{(E^2-EF-2F^2)-2S^2\}$	-1
X(32609)	$\{-3(E+8F)S^2+(7E+16F)S_BS_C$ $+8S_AS^2\}/6$	$\{(5E^2+10EF+32F^2)-16S^2\}/6$	4/3
X(34128)	$\{(7E-32F)S^2-(5E-4F)S_BS_C$ $-4S_AS^2\}/12$	$\{(3E^2-48EF-24F^2)+8S^2\}/12$	-1/3
X(34153)	$\{-3(E+8F)S^2+(5E+32F)S_BS_C$ $+8S_AS^2\}/4$	$\{(5E^2+4EF+80F^2)-16S^2\}/4$	2
X(36253)	$\{(5E-4F)S^2 –(5E+14F)S_BS_C$ $-6S_AS^2\}/4$	$-\{(E^2+26EF+52F^2)-12S^2\}/4$	-3/2
X(37477)	$\{(E-8F)S^2-(5E-4F)S_BS_C+4S_AS^2\}/2$	$\{(5E^2-14EF+8F^2)-8S^2\}/2$	2
X(37496)	$\{-(E-8F)S^2+(5E-4F)S_BS_C+4S_AS^2\}/6$	$\{3(E+10F)F-8S^2\}/6$	2/3
X(38626)	$\{(31E-32F)S^2-(49E-68F)S_BS_C$ $-36S_AS^2\}/8$	$-\{(5E^2+220EF-136F^2)-72S^2\}$ $/8$	-9/2
X(38632)	$\{-(23E+32F)S^2+(41E-4F)S_BS_C$	$\{(13E^2+140EF-8F^2)-72S^2\}/8$	9/2

	$+36S_AS^2\}/8$		
X(38633)	$\{3(7E-24F)S^2-(29E-88F)S_BS_C$ $-16S_AS^2\}/18$	$\{(5E^2-170EF+176F^2)+32S^2\}$ $/18$	-6/9
X(38638)	$\{-3(E+24F)S^2+(11E+56F)S_BS_C$ $+16S_AS^2\}/18$	$\{(13E^2-10EF+112F^2)-32S^2\}$ $/18$	8/9
X(40112)	$\{-(E+10F)S^2+6S_AS^2\}/3$	$\{(5E^2+EF-4F^2)-12S^2\}/3$	2
X(40113)	$\{-(5E+32F)S^2+(5E-4F)S_BS_C$ $+20S_AS^2\}/10$	$\{3(5E^2+6EF-8F^2)-40S^2\}/10$	2
X(40685)	$\{(11E-40F)S^2-9ES_BS_C-8S_AS^2\}/16$	$\{3(E^2-24EF-16F^2)+16S^2\}/16$	-1/2

From Table 4, X(74), X(265), X(3580), X(3581), X(9140), X(10264), X(10733), X(13455), X(14677), X(15055), X(15107), X(15360), X(20126), X(20127), X(25739) are on the line(1) parallel to the Euler line, and X(110), X(5655), X(7728), X(10540), X(10706), X(10721), X(12121), X(14157), X(22115), X(32111), X(34153), X(37477), X(40112), X(40113) are on the line(2) parallel to the Euler line. The line(1) and line (2) are on the opposite side of equal distance from the Euler line. Since the midpoints of the triangle centers on the line (1) and on the line (2) are on the Euler line. For example, **X(74) + X(110) = 2X(3),** since $x_e(74) + x_e(110) = \{-3(5E-4F)F + 4S^2 +(E^2 + 5EF+4F^2) - 4S^2\}/(E-8F) = (E^2-10EF +16F^2)/(E-8F) = (E-2F) = 2xe(3)$ (see Table 3). Similarly, X(113), X(1495), X(1511), X(1514), X(1531), X(1533), X(1539), X(1568), X(5642), X(10272), X(10564), X(11064), X(11693), X(11694), X(13202), X(13392), X(13851), X(13857), X(16163) are on the line (3) parallel to the Euler line, and X(125), X(10113), X(11801), X(12041), X(12295), X(15361), X(16111), X(21663), X(32269) are on the line (4). The line (3) and the line (4) are also on the opposite side of equal distance from the Euler line. Therefore, the midpoints of the triangle centers on the line (3) and those on the line (4) are on the Euler line. For example, **X(113) + X(125) = 2X(5),** since $x_e(113) + x_e(125) = \{(E^2+8EF-20F^2)-4S^2-12(E+F)F+4S^2\}/\{2(E-8F)\}= (E^2-4EF-32F^2)$ $/\{2(E-8F)\}=(E+4F)/2 = 2x_e(5)$(see Table 1). Further, the line (3) lies in the middle of the line (2) and the Euler line, and the line (4) on the middle of the line (1) and the Euler line. Therefore, (2 : 1) internally dividing points of the centers on the line (1) and those on the line (3) are on the Euler line. For example, since the Euler coordinates (xe, ye) for X(74)+2X(113) are given by $x_e = x_e(74) + 2x_e(113) = \{-3(5E-4F)F+4S^2+(E^2+8EF -20F^2) - 4S^2\}/(E-8F) = (E^2-7EF-8F^2)/(E-8F) = (E+F) = 3xe(2)$, and $ye^* = y_e(74)^* + 2y_e(113)^* = -2 + 2 = 0$, then, **X(74) + 2X(113) = 3X(2)**. Similarly, (2 : 1) internally dividing points of the centers on the line (2) and those on the line (4) are on the Euler line. For example, the Euler coordinates (xe, ye) for X(110) +2X(125) are xe =

$x_e(110)+2x_e(125)$ = {$(E^2+5EF+4F^2)-4S^2-6(E+F)F+4S^2$]/(E-8F) = $(E^2-7EF-8S^2)/(E-8F)$ = (E+F) and ye*= $y_e(110)^* + 2y_e(125)^*$ = 2 - 2= 0, **X(110) + 2X(125)=3X(2)** (see Table 1). Furthermore, (2 : 1) externally dividing points of the centers on the line (1) and those on the line (4) are on the Euler line. For example, the Euler coordinates (xe, ye) for -X(74) +2X(125) are x_e= - $x_e(74)+2x_e(125)$={$3(5E-4F)F-4S^2-12(E+F)F+4S^2$}/(E-8F)=3(E-8F)F /(E-8F) = 3F and $y_e^* = y_e(74)^* + 2y_e(125)^*$ = -2 + 2 = 0, then **-X(74)+2X(125) = X(4)** (see Table 1). Similarly, (2 : 1) externally dividing points of the centers on the line (2) and those on the line (3) are on the Euler line. For example, the Euler coordinates (x_e, y_e) for -X(110) + 2X(113) are x_e = $-x_e(110)+2x_e(113)$ = {$-(E^2+5EF+4F^2)+4S^2+(E^2+8EF-20F^2)-4S^2$}/(E-8F)= $(3EF-24F^2)/(E-8F)$ = 3F, ye* = - $y_e(110)^* + 2y_e(113)^*$ = -2 + 2 = 0, then **-X(110) + 2X(113)= X(4)** (see Table 1). Similarly many combos can be derived for the other centers in Table 4.